Chapter 4

Investment comparison

I) Present values, future values

Consider an actor A on the market. Assume that at time t it receives a cash flow a:

- $a > 0$ if he receives cash
- $a < 0$ if he pays cash

Consider a date $s < t$. Assume that we are given an interest rate r with compounding period 1 and that $t - s \in \mathbb{N}$. Then, the present value at time s of the future cash flow is $PV_0(a) = \underbrace{(1+r)^{034}}_{56\,0789:\,0345;} a$

The factor $(1+r)^{034}$ is called a discount factor.
Make the same assumptions as before, except that $s > t$.
Then, the cash flow a occurring at time t has a future value at time $s > t$, given by

$$FV_0(a) = \underbrace{(1+r)^{034}}_{=6\,0789:\,034=;} a$$

The factor $(1+r)^{034}$ is called an accumulation factor.

II) Criterion 1: present value of a sequence of cash flows

If we have an interest rate r playing the role of a reference, we can evaluate and compare investments using
Usually, one takes the riskless interest rate available on the market. There we assume that the compounding period for this rate r is 1.
The net present value at time 0 of a sequence of cash flow $\mathbb{C} = (c_;, c_6, \dots, c_B)$ occurring at times $0,1,..N$ is $NPV_E^;(C) = \sum_{4G;}^{B} (1+r)^{34} c_4$

This gives us a criterion for comparison: given two investments generating the sequences of cash flow $\mathbb{C}^{(6)}$, $\mathbb{C}^{(H)}$, we will prefer the first one if

$$NPV_E^;\left(\mathbb{C}^{6}_{(\,)}\right) > NPV_E^;\left(\mathbb{C}^{H}_{(\,)}\right)$$

If $NPV_E^;\left(\mathbb{C}^{6}_{(\,)}\right) = NPV_E^;\left(\mathbb{C}^{H}_{(\,)}\right)$, we say that $\mathbb{C}^{6}_{(\,)}$, $\mathbb{C}^{H}_{(\,)}$ are indifferent.

Limit of this criterion:

- It does not take risk into account
- Maturity should also play a role : for instance, if $NPV\left(\mathbb{C}^{(6)}\right) = NPV\left(\mathbb{C}^{(H)}\right)$, we should prefer the investment with smaller maturity N
- The interest rate is not easy to choose

III) The method of the yield rate

Definition: consider a sequence of cash flow $\mathbb{C} = (c_4)_{4G;,6,\dots,B}$. The yield rate, or Internal Rate of Return (IRR) of $\mathbb{C}$, is the interest rate $r^* \geq 1$ solution of $NPV_E^;(\mathbb{C}) = 0$

Note that $r^* = IRR(\mathbb{C})$ is intrinsic, it does not depend on another interest rate r.
Unfortunately, it can happen that $IRR(\mathbb{C})$ does not exist, or it is not unique.

Theorem: If a sequence of cash flows $\mathbb{C} = (c_6, \dots, c_B)$ is such that $c_6, c_H, \dots, c_M$ have the same sign, with at

leat one c_7 $(0 \le i \le p)$ non zero, for some $1 \le p \le n-1$.
Suppose in addition, that $c_{MT6}, c_{MTH}, \ldots, c_B$ all have the opposite sign to $(c_6, \ldots, c_M)$ and that at leat one c_U $(p+1 \le j \le N)$ is non zeo.

Then, the IRR of $\mathbb{C}$ exists and is unique.

→ *Proof*

We only write the proof when $c_6, \dots, c_M \leq 0$ and $c_{MT6}, \dots, c_B \geq 0$ (the other case follows, replacing $\mathbb{C}$ by $-\mathbb{C}$)

We will assume that for some $0 \leq p^{W} \leq p : c_{M^X} < 0$ and the case $p^{W} < p : c_{M^X T6} = \cdots = c_M = 0$.

Similarly, let $p^{WW} \geq p + 1$ such that $c_{M^{XX}} > 0$ and in the case $p^{WW} > p + 1 : c_{MT6} = \cdots = c_{M^{XX}36} = 0$.

In other words, p' is the last time t such that $c_4 < 0$ and p^{WW} is the first time t_M such that $c_4 > 0$.